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DAMPING OF SOUND WAVES BY BULK VISCOSITY IN REACTING GASES

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ABSTRACT

The very long-standing problem of sound waves propagation in fluids is reexamined. In particular, from the analysis of the wave damping in reacting gases following the work by Einstein (1929), it is found that the damping due to the chemical reactions occurs nonetheless the second (bulk) viscosity introduced by Landau and Lifshitz (1987) is zero. The simple but important case of a recombining Hydrogen plasma is examined.

Keywords: Viscosity, sound waves, damping, hydrogen plasma, reacting gas.

INTRODUCTION

Propagation of disturbances, in particular sound waves in hypothetical equilibrium fluids has been researched since the pioneer work by several famous researchers (Lord Rayleigh, 1964; Einstein, 1929; Einstein and de Donder, 1929; Lamb, 1932; Lighthill, 1978). The main characteristics of sound waves have been well established, i.e. the waves propagate with certain velocity and can be damped by the following irreversible processes: viscosities, thermal conduction, and chemical reactions. Landau and Lifshitz (1987) introduced a bulk (second) viscosity coefficient ζ in the equation of motion for accounting the dissipation of energy due to compression or expansion through transferring kinetic energy into internal degrees of freedom (such as chemical reactions, excitation of atomic/molecular levels, etc.). However, in the case of chemical reactions, such approximation only holds if one neglects any other effects except the density change $\delta\rho$ due to the chemical reaction.

Henceforth, as it will be shown at the present note, the Landau approximation is rather restrictive. In fact, if ξ is a parameter characterizing the degree of advance of chemical reaction in the fluid (say, the concentration of one chemical component) and ξ_0 its respective value at chemical equilibrium, which generally is a function of the equilibrium density ρ_0 and temperature T_0, say $\xi_0(\rho_0, T_0)$ (Vincenti and Kruger, 1975). Henceforth, as it can be realized, in the Landau approximation (Landau and Lifshitz, 1987; Ibáñez, 2009) the second viscosity coefficient is $\sim (\partial\xi_0/\partial\rho)_T$. Therefore, when $(\partial\xi_0/\partial\rho)_T = 0$, the acoustic wave damping is also zero. However, when $(\partial\xi_0/\partial T)_\rho \neq 0$, the sound waves could be damped nonetheless the Landau bulk viscosity coefficient is zero, as it will be shown below.

The present analysis on the bulk viscosity is made for any reacting gas where the chemical reactions can be reduced to a net reaction that can be described by one parameter measuring the advance of the reaction (Yoneyama, 1973; Ibáñez and Parravano, 1983). However, for context, the results are applied to a Hydrogen plasma where the simple reaction $H^+ + e^- \leftrightarrow H + (\chi)$ proceeds (χ being the ionization potential). The knowledge of the above plasma is of particular importance in Astrophysics, say, the solar atmosphere (Stein and Schwartz, 1972; Stein and Leibacher, 1974; Spitzer, 1978; Böhm-Vitense, 1987; Narain and Ulmschneider, 1990), the interstellar gas (Spitzer, 1978; Spitzer, 1982; Spitzer, 1990) and more recently in the Intracluster gas (Fabian *et al*., 2003; Ruszkowski *et al*., 2004; Fabian *et al*., 2005; Ferland *et al*., 2009), in particular due to the fact that wave dissipation has been invoked as one of the mechanisms of heat input. However, a detailed study of the thermal behavior of the above plasmas is out the scope of the present study, which is particularly restricted to find an expression of the bulk viscosity coefficient in chemically active plasmas.

Basic Equations

In general, for a one-dimensional (1-D) plane wave the wave number k and the angular frequency ω are related by:

$$k = \frac{\omega}{c} \qquad (1)$$

The parameter c is defined by the following relation:

$$c = \pm\sqrt{\frac{\partial p}{\partial \rho}} \qquad (2)$$

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where

$$\frac{\partial p}{\partial \rho} = \frac{1}{\rho_0}\left[p_0 - \frac{\partial (pV)}{\partial V}\right] \qquad (3)$$

with $V = 1/\rho$ and the equilibrium values denoted by the subscript "0".

Relation (1) formally obtained for nondispersive media also holds for dispersive media, for which c is a complex quantity (as well as k) (Landau and Lifshitz, 1987). Only for disturbances propagating in a nonreacting ideal fluid, relation (1) becomes the following adiabatic sound speed:

$$c = c_s = \sqrt{\left(\frac{\partial p}{\partial \rho}\right)_s} \qquad (4)$$

Strictly speaking, the basic gas dynamic equations admit solutions in the form $\sim \exp(\beta t + i\mathbf{k}.\mathbf{r})$ with $\beta = \sigma - i\omega$ and $\mathbf{k} = \mathbf{k}_r + i\mathbf{k}_i$, where σ and ω are real quantities and $\mathbf{k}_r$ and $\mathbf{k}_i$ are real vectors. Therefore, one may write the sound disturbance as $\sim \exp(\sigma t - k_i x) \exp[i(k_r x - \omega t)]$ for the one-dimensional problem. The above can be interpreted as a wave of frequency ω, wavelength $\lambda = 2\pi/k_r$, traveling along the x-axis with a phase velocity $v = \omega/k_r$ and the amplitude $\sim \exp(\sigma t) \exp(-k_i x)$. The first factor measures the attenuation (or growth if $\sigma > 0$) in time, and the second factor measures the spatial absorption (or amplification if $k_i < 0$) in ordinary progressive wave propagation studies (Markham *et al.*, 1951; Lifshitz and Pitaevskii, 1981; Ibáñez and Mendoza, 1987). The present analysis is restricted to the spatial absorption of linear wave propagation in a chemically active fluids from where the bulk viscosity coefficient is calculated.

For reacting gases, let's treat the set of "chemical reactions" being in progress that can be reduced to a single reaction described by the following equality: $\sum_j \nu_j A_j = 0$, where A_j are the chemical symbols of the reagents and the coefficients ν_j are positive or negative integers. For this treatment there is at least one component j, for which the concentration $\xi_j = n_j/n$ goes to zero when the reaction proceeds to a sense indefinitely, here n denotes the total number density of atoms and n_j is the number density for gas particles of the j^{th} component. So, one may introduce the parameters ξ and a such that

$$\xi_j = \frac{n_j}{n} = a\xi, \quad 0 \le \xi \le 1 \qquad (5)$$

where ξ denotes the degree of advance of the reaction and a is the maximum number of abundance ratio of the j^{th} component to the total number of nuclei.

With the equation of continuity for the different components and definition (5), one may obtain the following rate equation (Vincenti and Kruger, 1975; Yoneyama, 1973; Ibáñez and Parravano, 1983):

$$\frac{\partial \xi}{\partial t} + X(\rho, T, \xi(\rho, T)) = 0 \qquad (6)$$

where $X(\rho, T, \xi(\rho, T))$ is the net rate that can be at equilibrium $X(\rho_0, T_0, \xi_0(\rho_0, T_0)) = 0$.

Additionally, an ideal-like state equation will be assumed, i.e.

$$p = \frac{R\rho T}{\mu(\xi)} \qquad (7)$$

where R is the gas constant and $\mu(\xi)$ is the mean molecular weight, $\mu^{-1} = \Sigma_j \xi_j$.

On the other hand, the internal energy per unit mass becomes

$$u = A(\xi)RT + \chi N_0 a\xi \qquad (8)$$

where χ and N_0 denote the dissociation energy and the Avogadro's number, respectively, and

$$A(\xi) = \sum_j \frac{\xi_j}{\gamma_{j-1}} \qquad (9)$$

where γ_j is the specific heat-ratio for the j^{th} component.

For an adiabatic change, the energy equation can be written as follows:

$$RA(\xi)\frac{\delta T}{\delta t} - \frac{p}{\rho^2}\frac{\delta \rho}{\delta t} + RTB(\xi, T)\frac{\delta \xi}{\delta t} = 0 \qquad (10)$$

with

$$B(\xi, T) = \frac{dA}{d\xi} + \frac{a\chi}{k_B T} \qquad (11)$$

where k_B is the Boltzmann constant.

For linear disturbances close to the equilibrium, we get

$$RA_0 \delta T - \frac{p_0}{\rho_0^2}\delta \rho + RB_0 T_0 \delta \xi = 0 \qquad (12)$$

where $A_0 = A(\xi_0)$ and $B_0 = B(\xi_0, T_0)$ are the equilibrium values of the functions $A(\xi)$ and $B(\xi, T)$.

For fluctuations $\sim \exp(-i\omega t)$ from equation (6), it follows that the disturbances $\delta\xi$, $\delta\rho$, and δT are related by the following equation:

$$\delta \xi = \frac{\xi_\rho^*}{1 - i\omega\tau}\delta \rho + \frac{\xi_T^*}{1 - i\omega\tau}\delta T \qquad (13)$$

where $\tau = (\partial X/\partial \xi)^{-1}$ is the relaxation time that is a positive quantity for chemically stable gases. Also, $\xi_\rho^* =$

$(\partial\xi_0/\partial\rho)_T$ and $\xi_T^* = (\partial\xi_0/\partial T)_\rho$ are the corresponding derivatives at equilibrium (Yoneyama, 1973; Ibáñez, 2004; Ibáñez and Parravano, 1983).

Additionally, with equations (7), (6), and (12), equations (3) becomes

$$\frac{\partial p}{\partial \rho} = \frac{p_0}{\rho_0}[1 + Q] \qquad (14)$$

where the factor Q is given by

$$Q = \frac{(1-i\omega\tau)-(\mu_0 B_0+\mu_\xi A)\rho\xi_\rho^* - T\xi_T^*\mu_\xi/\mu}{\mu_0[A(1-i\omega\tau)+T\xi_T^* B_0]} \qquad (15)$$

In (15), μ_ξ is the derivative of the molecular weight with respect to the chemical parameter. It is important to mention that relation (14) for a particular simply chemical reaction was obtained in an early paper by Einstein (1929).

In the limiting when $\omega\tau \to \infty$ (frozen chemistry), $Q \to 1/A\mu_0$, and in the opposite limiting $\omega\tau \to 0$ (the chemical equilibrium follows the fluctuation) one can get

$$Q = \frac{1-(\mu_0 B_0+\mu_\xi A)\rho\xi_\rho^* - T\xi_T^*\mu_\xi/\mu}{\mu_0[A+T\xi_T^* B_0]} \qquad (16)$$

In the limiting case when the fluctuation $\delta\xi$ is only due to the change of density, $\xi_T^* = 0$, equation (15) reduces to

$$Q = \frac{(1-i\omega\tau)-(\mu_0 B_0+\mu_\xi A)\rho\xi_\rho^*}{\mu_0 A(1-i\omega\tau)} \qquad (17)$$

On the opposite limit when $\xi_\rho^* = 0$, we get

$$Q = \frac{(1-i\omega\tau)-T\xi_T^*\mu_\xi/\mu}{\mu_0[A(1-i\omega\tau)+T\xi_T^* B_0]} \qquad (18)$$

If in the limiting $\xi_T^* = 0$ additionally $\xi_\rho^* = 0$, henceforth $Q = 1/A\mu_0 = \gamma - 1$. Therefore, from equation (14) $\sqrt{\partial p/\partial\rho} = \sqrt{\gamma p_0/\rho_0}$ (being γ the specific heat ratio) becomes the isentropic sound speed c_s^2 in a nonreacting ideal gas, as it should be.

It is interesting to point out that in the Landau approximation (Landau and Lifshitz, 1987), where the fluctuation $\delta\xi$ is assumed to occur at a constant entropy S, i.e. the change of pressure p is due only to the change of density $\delta\rho$ produced by the fluctuation in the chemical parameter $\delta\xi$, one gets

$$\frac{\partial p}{\partial \rho} = \frac{1}{1-i\omega\tau}[c_0^2 - i\omega\tau c_\infty^2] \qquad (19)$$

where c_0^2 and c_∞^2 are given by

$$c_0^2 = \left(\frac{\partial p}{\partial \rho}\right)_{eq} = \left(\frac{\partial p}{\partial \rho}\right)_\xi + \left(\frac{\partial p}{\partial \xi}\right)_\rho \left(\frac{\partial \xi_0}{\partial \rho}\right), c_\infty^2 = \left(\frac{\partial p}{\partial \rho}\right)_\xi \qquad (20)$$

From (15), one obtains the corresponding parameter Q_L in the Landau approximation, i.e.

$$Q_L = -\frac{1}{1-i\omega\tau}\frac{\mu_\xi}{\mu_0}\rho\xi_\rho^* \qquad (21)$$

Finally, in the limiting case when $\xi_\rho^* = 0$, it follows that $\sqrt{\partial p/\partial\rho} = \sqrt{p_0/\rho_0}$, i.e. the sound propagation would occur with the isothermal sound speed as it is expected. Additionally, at the Landau's approximation the effects of the chemical reaction may be accounted for introducing the second viscosity coefficient in the motion equation given by the following expression:

$$\zeta = \frac{\rho_0\tau}{1-i\omega\tau}[c_\infty^2 - c_0^2] = \frac{\rho_0\tau}{1-i\omega\tau}\frac{p_0\mu_\xi}{\mu_0}\xi_\rho^* \qquad (22)$$

Equation (22) represents the Landau bulk viscosity coefficient ζ [g×cm^{-1}×s^{-1}], as it can be readily verified with equation (20).

Collisionally Ionized Hydrogen Plasma

For context, at the present section the above results will be applied to the simple but important examples of an ionized Hydrogen gas when it is collisionally ionized. As it will be shown the damping of sound waves becomes zero at the Landau approximation but does not equal to zero at the Einstein approximation.

A collisionally ionized Hydrogen plasma can be considered as a reacting plasma, where the following reaction

$$H^+ + e^- \leftrightarrow H^0 + \chi \qquad (23)$$

proceeds with the following expressions

$$A = \frac{1}{(\gamma-1)\mu}, B_0 = \frac{1}{\gamma-1} + \frac{\chi}{k_B T}, \mu = \frac{1}{1+\xi} \qquad (24)$$

In (24), ξ is the degree of the ionization, χ the Hydrogen ionization potential, and k_B the Boltzmann constant. The subscript "0" indicates equilibrium values that have been omitted. Additionally, the generalized ionization recombination rate function (Yoneyama, 1973; Ibáñez and Parravano, 1983) becomes equal to

$$X = N_0\rho\alpha(T)\xi^2 - N_0\rho q(T)\xi(1-\xi) = 0 \qquad (25)$$

Therefore, at equilibrium there is

$$\xi^*(T) = \frac{q(T)}{\alpha(T)+q(T)} \qquad (26)$$

where the total recombination coefficient $\alpha(T)$ and the collisional ionization rate $q(T)$ are respectively given by

$$\alpha(T) = \frac{2.06\times10^{-11}}{T}\left(0.5\ln\Theta + \frac{0.47}{\Theta^{1/3}} - 0.32\right)\left[\frac{cm^3}{s}\right] \quad (27)$$

$$q(T) = 5.85\times10^{-11}\sqrt{T}\exp(-\Theta)\left[\frac{cm^3}{s}\right] \quad (28)$$

In (28), $\Theta = 1.579 \times 10^5/T$ (Seaton, 1959; Hummer and Seaton, 1962; Hummer, 1962). The above approximation holds (Parker, 1953; Corbelli and Ferrara, 1995) in the range of $3.5 \times 10^3 \leq T(K) \leq 1.58 \times 10^5$.

For a collisionally ionized Hydrogen plasma, from equation (26) follows that $\partial\xi_0^*/\partial\rho = 0$. Therefore, the second viscosity in the Landau approximation, equation (22), is also equal to zero. From equation (14) however, the speed of sound c becomes

$$c = \sqrt{\frac{p(1+Q)}{\rho}} \quad (29)$$

with

$$Q = \frac{(\gamma-1)[1-\mu T\xi_T/(1-i\omega\tau)]}{1+(\gamma-1)B\mu T\xi_T/(1-i\omega\tau)} \quad (30)$$

This means that the damping effect occurs due to the irreversible process inherent to the chemical reaction, as it follows from the fact than c becomes a complex quantity as well as the wave number k in equation (1): $k = k_r + ik_i$, where k_r and k_i are real quantities. Here $k_i > 0$ (Ibáñez, 2004) represents the damping coefficient.

On the other hand, from equations (25)-(28) the relaxation time becomes

$$\tau = \frac{1}{N_0\rho q(T)} \quad (31)$$

i.e. the relaxation time $\tau > 0$ and therefore, the Hydrogen plasma is chemically stable.

The damping per unit wave length $2\pi k_i/k_r$ and the phase velocity v_{ph}/c_T normalized to the isothermal sound speed $c_T = (p_0/\rho_0)^{1/2}$ have been plotted in Figures 1a and 1b, respectively, as functions of temperature T for three different values of $\omega\tau$ (10^{-1} for the dash line, 1 for the thick line, and 10 for the dotted line). Regardless the value of $\omega\tau$ the damping shows maxima, and the phase speed shows minima at a temperature close to $\log(T) = 4.16$, i.e. the temperature, at which the function ξ_T^* becomes a maximum and the effect of the recombination-ionization process become important. At very low (neutral Hydrogen) as well as at very high temperatures (ionized Hydrogen), the damping tends to be zero (Fig. 1a), and the sound velocity tends to be the isentropic sound speed (Fig. 1b), as it is expected from simple physical considerations.

In Figures 1c and 1d the damping per unit wave length ($2\pi k_i/k_r$) and the normalized phase velocity (v_{ph}/c_T) are respectively plotted but as functions of $\omega\tau$ for temperatures slight lower ($\log(T) = 4.04$, (dash line) and higher $\log(T) = 4.28$, (dotted line) than $\log(T) = 4.16$ (thick line). The damping per unit wave length becomes a maximum very close to the value of $\omega\tau$ (Fig. 1c) where the inflexion point of v_{ph}/c_T occurs as shown in figure 1d. Regardless of the temperature value, waves with $\omega\tau \ll 1$ propagate as adiabatic disturbances in a gas at chemical equilibrium, and those with $\omega\tau \gg 1$ as adiabatic disturbances in a gas, where the chemical reaction is frozen. In the above limiting cases the disturbances tend to be undamped waves (Fig. 1c) as it should be.

Photo-Ionized Hydrogen Plasma

In this section, the results of the previous section will be applied to a photo-ionized Hydrogen plasma model i.e. an optically thin Hydrogen plasma ionized by a background radiation field of averaged photon energy E and photo-ionization rate ς. The net rate function $X(\rho, T, \xi)$ present in equation (6) can be expressed as follows:

$$X(\rho,T,\xi) = N_0\rho[\xi^2\alpha - (1-\xi)\xi q] - (1-\xi)(1+\phi)\varsigma \quad (32)$$

In (32), α is the total recombination coefficient [cm^3s^{-1}] that is given in equation (27), q is the collisional ionization rate [cm^3s^{-1}] according to Black (1981). Also, $\phi(E, \xi)$ is the number of secondary electrons that is in general a function of the mean photon energy E and the ionization ξ (Shull and Van Steenberg, 1985) and ς is the photo-ionization rate [s^{-1}] (Black, 1981). The last term of the right-hand side in equation (32) just accounted for this effect. Therefore, the corresponding terms in the energy equation (10) have to be added for consistency in order to take into account the heat input and output of energy by radiation. So, instead of equation (10) one obtains

$$RA(\xi)\delta T - \frac{p}{\rho^2}\delta\rho + RB(T,\xi)T\delta\xi + \delta L(\rho,T,\xi) = 0 \quad (33)$$

where the net heat/cooling function becomes

$$L(\rho,T,\xi) = N_0\{\rho^2[(1-\xi)\xi\Lambda_{eH}] + \xi^2\Lambda_{eH^+}\} - N_0\{(1-\xi)[E_h + (1+\phi)\chi]\}$$

where Λ_{eH} and Λ_{eH^+} are the cooling losses by $e{-}H$ and $e{-}H^+$ collisions (Ibáñez, 2004) neglecting secondary electrons $\phi = 0$.

According to Shull and Van Steenberg (1985), $0.002 \leq \phi \leq 0.366$ for $0.95 \geq \xi \geq 10^{-4}$ the exact value depends on E (it depends on the particular optical depth in the gas) and strictly speaking a self-consistent radiative transfer problem should be worked out. However, this is out of the scope of the present paper, whose aim is restricted to obtain an indicative value of the bulk viscosity for a photo-ionized Hydrogen plasma. Therefore, if in a first approximation the production of secondary electrons is neglected ($\phi = 0$), from equation (32) an explicit form of the ionization $\xi^*(\rho, T)$ at equilibrium can be obtained, i.e.

$$\xi^*(\rho,T) = \frac{N_0\rho q - \varsigma + \sqrt{B_p}}{2N_0\rho(\alpha+q)} \qquad (34)$$

with

$$B_p = (N_0\rho q + \varsigma)^2 + 4N_0\rho\alpha\varsigma \qquad (35)$$

Otherwise the solution for ξ at equilibrium becomes an implicit function of T and ρ, and for its calculation one must proceed numerically. The correction introduced by the secondary electrons is equivalent to an increase of the value of the photo-ionization rate, as it can be verified from equation (32).

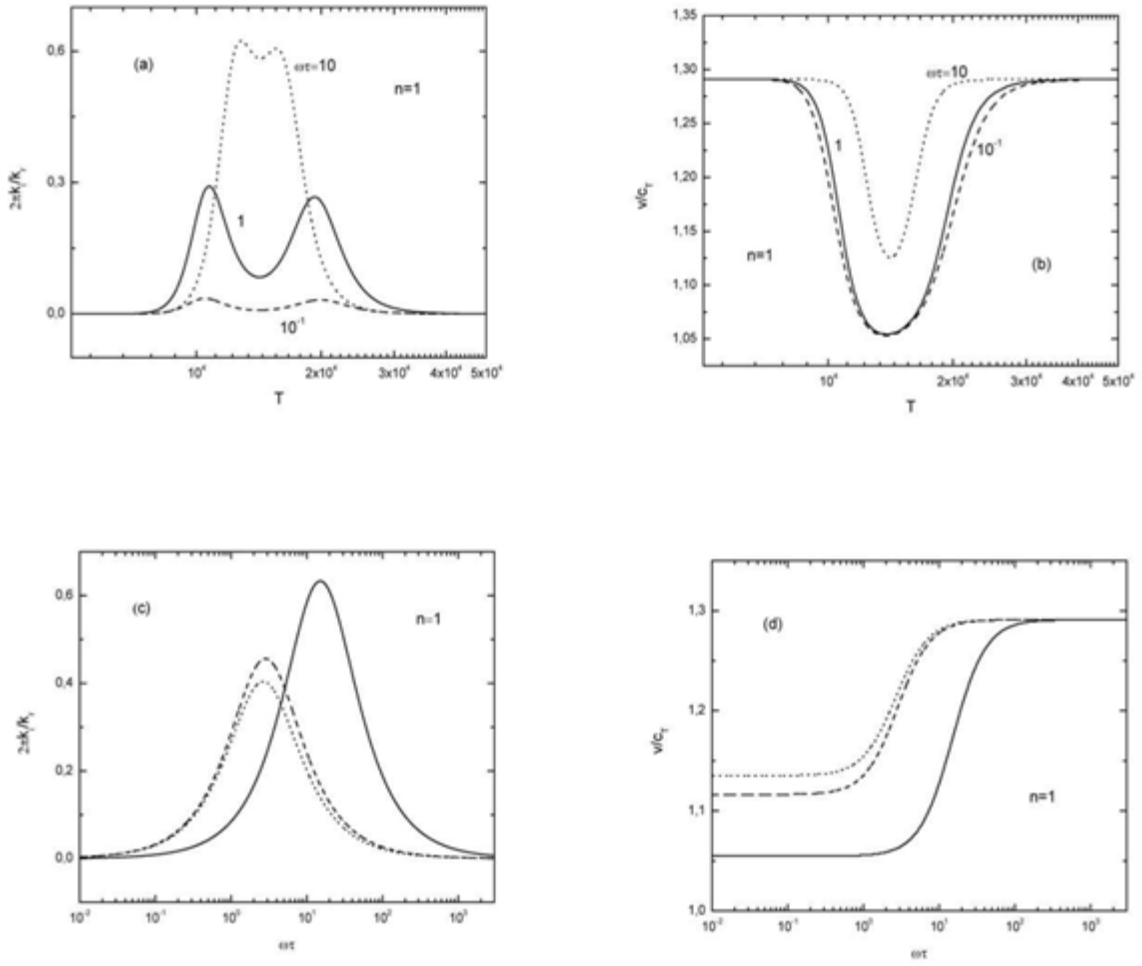

Fig. 1. (a) The damping per unit wave length $2\pi k_i/k_r$ as a function of temperature for three different values of the dimensionless frequency $\omega\tau = 10^{-1}$ (dash line), 1 (thick line), and 10 (dotted line). (b) The phase velocity v_{ph}/c_T normalized to the isothermal sound speed $c_T = \sqrt{p_0/\rho_0}$ as a function of temperature T for three different values of the dimensionless frequency $\omega\tau = 10^{-1}$ (dash line), 1 (thick line) and, 10 (dotted line). (c) The value of $2\pi k_i/k_r$versus $\omega\tau$for three different values of the temperature $\log(T) = 4.04$ (dash line), $\log(T) = 4.16$ (thick line) and $\log(T) = 4.28$, dotted lines). (d) The normalized velocity v_{ph}/c_T versus $\omega\tau$ for three different values of the temperature $\log(T) = 4.04$ (dash line), $\log(T) = 4.16$ (thick line), and $\log(T) = 4.28$ (dotted lines).

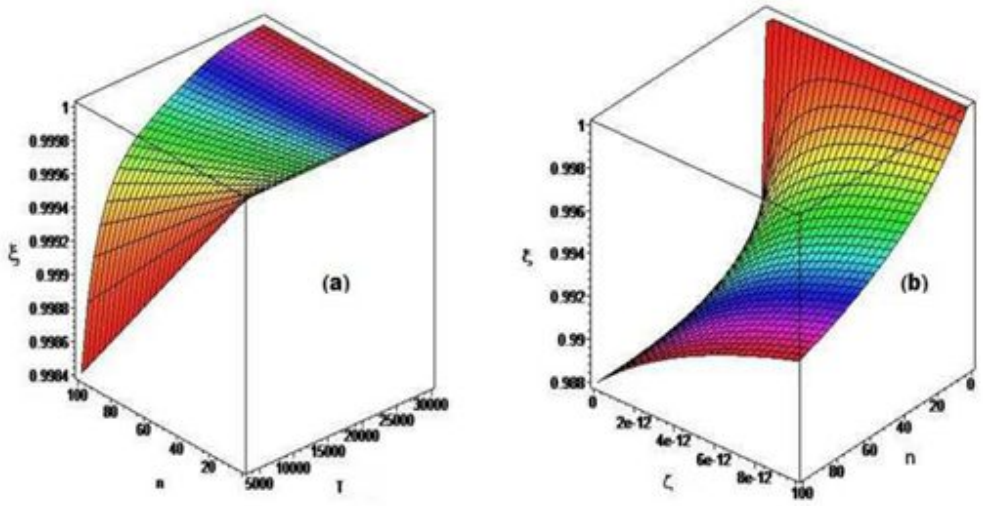

Fig. 2. (a) The equilibrium ionization ξ^* versus both the temperature T [K] and density n [cm^{-3}] for a photo-ionization rate $\varsigma = 5 \times 10^{-13}$ [s^{-1}]. Here, the red and magenta colors refer to the $T \rightarrow 5{,}000K$ and to the highest $T \rightarrow 30{,}000K$, respectively. (b) The 3D plot for the equilibrium ionization ξ^* versus both the density n [cm^{-3}] and the ionization rate ς [s^{-1}] for a fixed value of temperature, $\log(T) = 4.16K$. Here the color palette indicates the values of the density n: the red and magenta colors refer to $n \sim 0$ and $n \rightarrow 100$ [cm^{-3}], respectively.

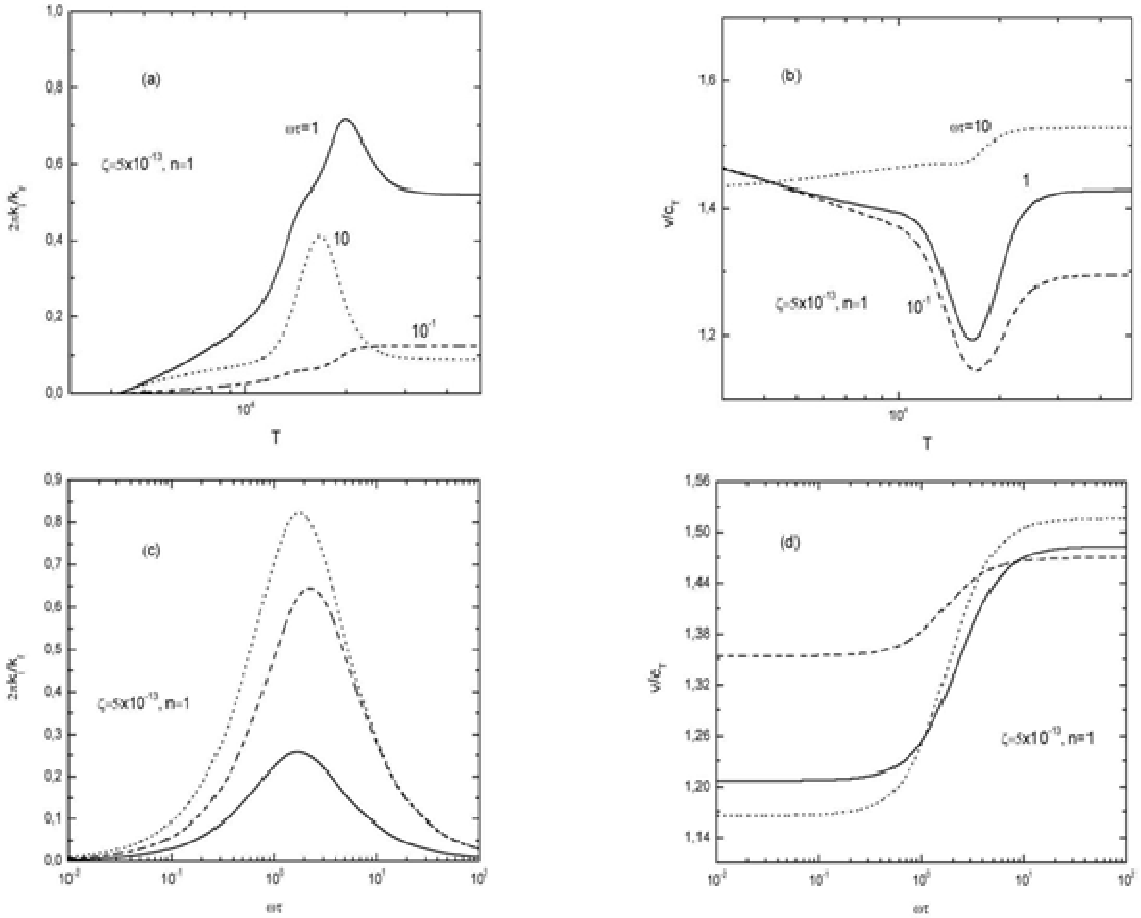

Fig. 3. (a) As in Figure 1a but for the photo-ionized gas with the rate given by expression (32) and $\varsigma = 5 \times 10^{-13}$ [s^{-1}]. (b) As in Figure 1b but for the photo-ionized gas with the rate given by expression (32) and $\varsigma = 5 \times 10^{-13}$ [s^{-1}]. (c) As in Figure 1c but for $\varsigma = 5 \times 10^{-13}$ [s^{-1}]. (d) As in Figure 1d but for $\varsigma = 5 \times 10^{-13}$ [s^{-1}].

Therefore, from equation (34) one obtains

$$\xi_\rho^* = \frac{\rho B_{p\rho} + 2\varsigma\sqrt{B_p} - 2B_p}{4\sqrt{B_p}N_0\rho^2(\alpha+q)} \qquad (36)$$

and

$$\xi_T^* = \frac{B_{pT}(\alpha+q) - 2B_p(\alpha_T+q_T)}{4\sqrt{B_p}N_0\rho(\alpha+q)^2} + \frac{2N_0\rho\sqrt{B_p}[\alpha q_T - q\alpha_T + \bar{\varsigma}(\alpha_T+q_T)]}{4\sqrt{B_p}N_0\rho(\alpha+q)^2} \qquad (37)$$

Here $B_{p\rho} = \partial B_p/\partial\rho$ in equation (36), and $B_{pT} = \partial B_p/\partial T$, $\alpha_T = \partial\alpha/\partial T$, $q_T = \partial q/\partial T$ in equation (37). Similar to the previous section, from equations (1), (14), and (15) one may calculate both the real and imaginary parts of c and k. However, for this particular plasma ξ^* is a function of both ρ and T instead of only T as given by equation (26).

Figure 2a shows the 3D plot of the ionization rate ξ^* as the function of $T(K)$ and density n [cm^{-3}]. In figure 2a, the red color refers to temperatures close to 5,000K. On the other hand, the magenta color gives the highest temperatures that are of the order of 30,000K for a fixed value of the photo-ionization $\varsigma = 5 \times 10^{-13}$ [s^{-1}]. Figure 2b shows the 3D plot of the ionization ξ^* as a function of the density n [cm^{-3}] and the photo-ionization ς [s^{-1}] for a fixed value of temperature ($\log(T) = 4.16K$), spanning in the range of values for the galactic interstellar medium (Klessen and Glover, 2014). In Figure 2b, the color indicates the values of the density: the red color refers to the densities n near zero values and the magenta color indicates the values of the density n close to 100 [cm^{-3}]. From both Figures 2a and 2b follows that the effect of the ionizing radiation increases the ionization at any temperature, respect to that resulting by collisions only. However, the strong ionization occurring at temperature $T \approx 2 \times 10^4 K$ is determined by collisions for galactic values of the photo-ionization rate ς [s^{-1}].

As it can be verified, the presence of the ionizing radiation field shifts the value of ξ_T^* towards higher temperatures ($\log(T) = 4.21$ for $\varsigma = 5 \times 10^{-13}$ [s^{-1}]) and smooths the change in the damping per unit wave length with the temperature T for any wave frequency. This can be found by comparing Figure 1a with Figure 3a, in which the damping $2\pi k_i/k_r$ is plotted as the function of T for $\varsigma = 5 \times 10^{-13}$ [s^{-1}]. Figure 1a shows the same three values of $\omega\tau$ but for the rate given in equation (32) instead of (25). The change of value of the maxima of the damping per unit wave depends on the value of $\omega\tau$. In particular it increases for $\omega\tau = 1$. Additionally, they are shifted towards the higher values of T following the shift of the maximum of ξ_T^* as it follows from the physical considerations.

Accordingly, the produced change of the phase velocity (taking into account the photo-ionization) can be seen when Figure 3b is comparing with Figure 1b. The minimum is shifted towards higher temperatures. However, Figure 3b demonstrates the smoother dependence at high frequencies ($\omega\tau$) as it can be seen when the dotted lines ($\omega\tau = 10$) in the two Figures mentioned above are compared.

At a particular temperature, the changes in the damping per unit wave length (therefore in the phase velocity, too) are small for galactic values of the photo-ionization ς. The reader can also find it when Figure 3c is compared with Figure 1c and Figure 3d is compared with Figure 1d. Generally, the qualitative and quantitative effects of the photo-ionization are small respect to those produced by collisions only in an atomic Hydrogen gas, as far as the sound wave propagation is concerned, and in the range of values of the parameters considered above.

Physical Implications

The aim of this section is to compare the values of the three absorption coefficients corresponding to: the bulk viscosity $\tilde{k}_b = c_T k_i/\omega$, the dynamical viscosity $\tilde{k}_v$, and the thermal conduction $\tilde{k}_\kappa$ that are given by (Lifshitz and Pitaevskii, 1981; Landau and Lifshitz, 1987). The dynamical viscosity $\tilde{k}_v$ is

$$\tilde{k}_v = \frac{2\omega v}{3c_T^2\gamma^{3/2}} \qquad (38)$$

where v is the kinematic viscosity.

The thermal conduction $\tilde{k}_\kappa$ is defined by

$$\tilde{k}_\kappa = \frac{\omega(\gamma-1)\chi}{2c_T^2\gamma^{3/2}} \qquad (39)$$

where χ corresponds to the thermometric conductivity (Parker, 1953; Spitzer, 1962; Braginskii, 1965; Lifshitz and Pitaevskii, 1981).

The problem of sound wave propagation in a self-consistent model of the atomic gas in the galaxy and other plasmas represents a great interest in Astrophysics. For this problem, H_e and ions of H_e, and ions of heavy elements can be included that will be published elsewhere in the future.

Incidentally there is another irreversible process in plasmas due to the frictional force between ions of mass m_i (and velocity v_i), and neutral particles of mass m_n (and velocity v_n) (Braginskii, 1965). The time scale for equalizing the velocities can be easily calculated from the respective Braginskii relations, from which one obtains the following equation.

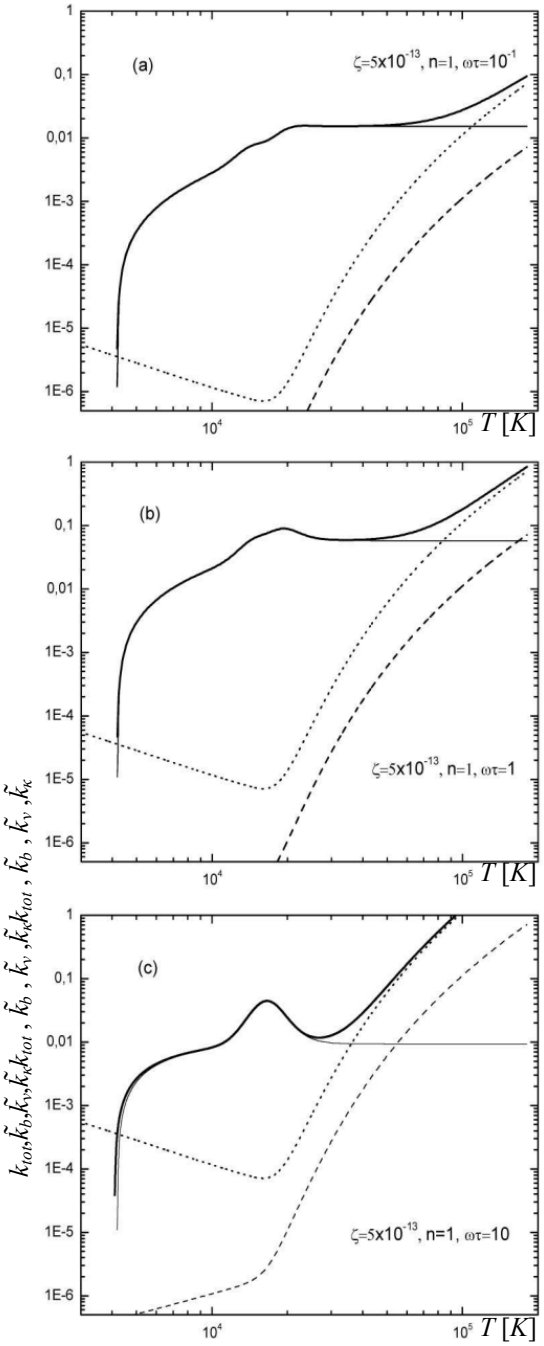

Fig. 4. The absorption coefficients $\tilde{k}_b$ (thin line), $\tilde{k}_v$ (dash), $\tilde{k}_\kappa$ (dotted), and the total absorption (thick) $k_{tot} = \tilde{k}_b + \tilde{k}_v + \tilde{k}_\kappa$versus the temperature T for $n = 1$ [cm^{-3}], the photo-ionization rate $\varsigma = 5 \times 10^{-13}$ [s^{-1}] when (a) the frequency $\omega\tau = 10^{-1}$, (b) $\omega\tau = 1$, (c) $\omega\tau = 10$.

$$\tau_{ni} \approx \frac{(m_i+m_n)}{\langle\sigma v\rangle(\rho_i+\rho_n)} \qquad (40)$$

where $<\sigma v>$ is the mean value of the product of the cross-section and the relative velocity averaged over all velocities. As it can be easily verified, generally $\tau_{ni} << \tau$. Additionally, the frictional damping becomes independent on the wave-length λ, and it is only important for oscillations with very high frequencies, and in plasmas with very low ionization (Braginskii, 1965; Nomura *et al*., 1999; Watson *et al*., 2004). Such effect will not be considered at the present discussion.

Figures 4a, 4b, and 4c show the plots of the absorption coefficients $\tilde{k}_b$ (thin line), $\tilde{k}_v$ (dash line), $\tilde{k}_\kappa$ (dotted line), and the total absorption $k_{tot} = \tilde{k}_b + \tilde{k}_v + \tilde{k}_\kappa$ (thick line) in units of [cm^{-1}]. All these coefficients depend on the temperature T for $n = 1$ [cm^{-3}]. The photo-ionization rate value is $\varsigma = 5 \times 10^{-13}$ [s^{-1}] and three different values of the dimensionless frequency are $\omega\tau = 10^{-1}$, 1, and 10. The relaxation time $\tau = |X_\xi|^{-1}$ [s] was also studied for $n = 1$ and the following three values of the photo-ionization rate: $\varsigma = 5 \times 10^{-14}$, 5×10^{-13}, and 10^{-12} [s^{-1}]. However, these results for the graphical study of the relaxation time τ are not given in this paper. Due to the fact that the effect of damping of sound waves is linear, it is worthy to calculate the total absorption coefficients due to the aforementioned three effects.

The absorption by the bulk viscosity becomes dominant in the following range of temperatures where the recombination-ionization takes place: $4.2 \times 10^3 \leq T \leq T_M(\omega\tau)$. Here the function of the wave frequency $T_M(\omega\tau)$ increases when $\omega\tau$ decreases as it is shown in Figures 4a, 4b, and 4c. At high temperatures ($T > T_M$) and high ionization, the thermal conduction (by electrons) dominates. In contrast, at low temperatures $T \leq 4.2 \times 10^3 K$, the thermal conduction by neutral atoms becomes dominant. At frequencies $\omega\tau \geq 1$ the bulk viscosity coefficient shows a conspicuous (relative) maximum. On the other hand, the dynamical viscosity is significantly lower (more than one order of magnitude) than both the bulk viscosity and the thermal conduction in the range of temperature under the consideration. In conclusion, in the photo-ionized Hydrogen plasma, the bulk viscosity is the most important damping mechanism in the aforementioned range of temperature: $4.2 \times 10^3 \leq T \leq T_M(\omega\tau)$.

CONCLUSION

Following the work by Einstein (1929) on propagation of sound waves in reacting gases, the bulk viscosity coefficient introduced by Landau and Lifshitz (1987) (equation (22) has been generalized to chemically active gases. It was found that the bulk viscosity coefficient becomes the imaginary part of the wave vector. In particular, for a collisionally ionized Hydrogen gas, the bulk viscosity in the Landau approximation becomes zero. However, it is different from zero at the present approximation, see the results obtained in the previous

sections. For context, additionally the bulk viscosity is also calculated for the photo-ionized Hydrogen gas for the values of the characteristic parameters of the high latitude atomic gas in the Galaxy.

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